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Resonance and trapping of topographic transient ocean waves generated by a moving atmospheric disturbance

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Proudman resonance amplifies the oceanic forced wave beneath moving atmospheric pressure disturbances. The amplification varies with water depth; consequently, the forced wave beneath a disturbance crossing topography radiates transient free waves. Transients are shown to magnify the effects of Proudman resonance for disturbances crossing the coast or shelf at particular angles. A Snell like reflection law gives rise to a type of resonance for relatively slow moving disturbances crossing a coast in an otherwise flat-bottomed ocean. This occurs for translation speeds less than the shallow water wave speed for disturbances approaching the coast at a critical angle given by the inverse sine of the Froude number of the disturbance. A disturbance crossing the shelf at particular angles can also excite seiche modes of the shelf via generation of a transient at the continental slope. Beyond a typically small angle of incidence, transients generated by a disturbance crossing the continental slope and coast will be trapped on the shelf by internal reflection. The refraction law for a fast-moving forced wave crossing an ocean ridge at greater than a small angle of incidence also results in trapped free-wave transients with tsunami-like periods propagating along the ridge. The subcritical resonance, excitation of shelf modes and trapping of the transients may have implications for storm surges and the generation of destructive meteotsunami.

1. Introduction

Disturbances in atmospheric pressure due to storms or fronts force sea-level variations of $O(10\,\mathrm{cm})$ due to the inverted barometer effect (Doodson 1924). Under moving disturbances, this effect is amplified by Proudman resonance (Lamb 1932; Proudman 1953). The amplification of the forced wave in a flat-bottomed ocean becomes large as the translation speed of the disturbance approaches the shallow water wave speed. The amplitude of the forced wave due to a disturbance travelling parallel to the coast over a linearly sloping bottom can also become large because of Greenspan resonance, which occurs for translation speeds near one of the coastally trapped edge-wave modes (Greenspan 1956). Both Proudman and Greenspan resonances give large responses at critical translation speeds corresponding to particular wave speeds. Here a resonance is shown to occur when a relatively slow moving forced wave in a flat-bottomed ocean crosses a vertical coast at a particular angle. This resonance occurs at translational speeds less than the shallow water wave

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speed, i.e. at subcritical speeds, and may be a new example of 'classic' resonance in the sense that it results from linear barotropic wave dynamics.

Garrett (1970) showed how free waves are generated when an atmospheric disturbance crosses a step. Garrett developed this generation mechanism as an explanation for why tide gauges recorded disturbances because of the Krakatoa eruption before oceanic waves could have arrived at San Francisco by the shortest route, speculating this may be due to focusing of the free waves generated when the atmospheric blast wave due to the eruption crossed the Aleutian and Hawaiian ridges. The extremely high propagation speeds for the atmospheric wave, 300 $\mathrm{m\,s^{-1}}$, exceeded the shallow water wave speed on both the deep and shallow sides of the step. Garrett (1970) noted the possibility of typically slow moving weather crossing ridges and creating free waves, commenting that they would be too small to be detected, though Vennell (2007) (hereafter V07) showed how, for less common fast storms, the transients can have significant amplitudes on shelves and ridges. Garrett (1970) focused on supercritical disturbances and did not explore the consequences for subcritical disturbances crossing a step nor looked at the limit of reflection off a vertical coast, which, as shown here, leads to a resonance.

V07 discussed how the forced oceanic wave beneath a small fast-moving storm radiates long free-wave transients as it crosses the coast, continental slope or a ridge at right angles. Transients reflected back into deep water are small, but those transmitted into shallow water are significant, comparable in size to the enhanced forced wave in shallow water due to Proudman resonance. Also, V07's numerical results demonstrated that a gentle continental slope, ten storm widths wide, did not significantly reduce the amplitudes of the transients transmitted onto a shelf or a ridge, when compared with those for a step. These linear free waves, referred to as topographic transients, had periods comparable with the passage time of the storm. This could be under an hour for small fast storms, e.g. storms traversing the Grand Banks with translation speeds of up to 30 $\mathrm{m\,s^{-1}}$, which is comparable to the shallow water wave speed over the Banks (Mercer *et al.* 2002).

When a free wave crosses a step into shallower water, refraction decreases the propagation angle of the transmitted wave. In the absence of Coriolis effects, this makes it unlikely that the energy of a free wave crossing onto a flat shelf or a ridge with straight parallel depth contours can be trapped in shallow water by internal reflection, though it can be trapped or focused by curved topography (Longuet-Higgins 1967). In contrast, refraction increases the propagation angle of the transmitted transient wave generated by a subcritical speed forced wave crossing from deep to shallow water. This opens up the possibility of trapping the topographic transients generated by a disturbance crossing a shelf or a ridge, which have periods too short for Coriolis effects to be important. The trapped, transmitted and reflected transients generated by a disturbance crossing the continental slope and coast may contribute to relatively high frequency sea-level variability at locations far from where a disturbance crossed the coast and may also excite resonant modes of shelves or inlets.

In this paper, it is shown that the transients generated at a vertical coast or step by an obliquely incident atmospheric pressure disturbance may enhance coastal surges under some storms, excite shelf seiches and may also contribute to understanding how the effects of seemingly weak pressure fronts can be amplified to give large waves which can result in destructive meteotsunami events at several locations around the globe (Rabinovich & Monserrat 1998; Vilibić & Mihanović 2003; Monserrat, Vilibic & Rabinovich 2006). For example, bursts of small atmospheric pressure variations of around 2–3 hPa propagating across Menorca Island at 20–30 m s^{-1} can excite a 10 min period seiche with an amplitude of more than 1 m within Ciutadella inlet (Rabinovich & Monserrat 1998). Specifically, this paper examines how the incident angle of the forced wave determines the amplitudes and fates of the transients generated by a disturbance crossing a coast, shelf or ridge. The paper uses deliberately simple techniques, such as ray tracing, in order to clearly understand how reflection and refraction govern the fates of the transients. Importantly, in two cases transients are shown to magnify the effects of Proudman resonance for disturbances crossing the coast or shelf at particular angles. The structure of this paper is as follows. Section 2 develops the linear forced shallow water equations and gives solutions for a forced wavefront crossing a coast and a step. The discussion explores a range of consequences of the solutions, subcritical resonance, trapping of transients on a shelf or a ridge, excitation of shelf modes, the effects of a finite breadth forced wavefront as well as the potential impact on coastal storm surge.

2. Model solutions

A simple two-dimensional barotropic model for transient shallow water waves generated by moving pressure disturbances over topography is developed. The moving disturbance's time scale is $T_p = L/U$, the time taken for it to pass a fixed observer, where L is the width of the disturbance and U is its translation speed. This time scale is assumed to be large enough that motions are governed by shallow water dynamics, i.e. longer than far-infra gravity waves, but small compared to the earth's rotational period so that Coriolis effects can be neglected. The time scale falls in the band where bottom friction can be neglected at zeroth order. In shallow water, the hydrostatic balance gives the pressure at any depth $-z$ as $p = \rho g(\eta - z) + p_a(x, y, t)$, where η is the displacement of the ocean's surface and $p_a(x, y, t)$ is the atmospheric pressure at the ocean's surface due to the disturbance. The atmospheric pressure can be defined in terms of the inverted barometer response as $p_a = -\rho g \eta_a$, where η_a is the ocean's surface displacement under a stationary disturbance. A 1 hPa change in air pressure gives rise to approximately a 1 cm change in sea level (Doodson 1924). Neglecting rotational and bottom frictional effects, the linearized atmospherically forced barotropic two-dimensional equations for shallow water motion in constant water depth can be expressed as (Gill 1982)

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x} + g\frac{\partial \eta_a}{\partial x} + \frac{\tau^x}{\rho h}, \tag{2.1}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y} + g\frac{\partial \eta_a}{\partial y} + \frac{\tau^y}{\rho h}, \tag{2.2}$$

$$\frac{\partial \eta}{\partial t} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \tag{2.3}$$

where u and v are the velocity components, (τ^x, τ^y) are the components of wind stress and η is assumed small compared to the water depth h. Combining (2.1)–(2.3) for constant water depth gives the linear forced two-dimensional wave equation

$$\frac{\partial^2 \eta}{\partial t^2} - c^2\nabla^2\eta = -c^2\nabla^2\eta_a + \frac{1}{\rho}\nabla\cdot\tau = -c^2\nabla^2 F, \tag{2.4}$$

where $c=\sqrt{gh}$ and $\nabla^2=(\partial^2/\partial x^2)+(\partial^2/\partial y^2)$. For convenience, the atmospheric pressure forcing and the divergence of the wind stress have been combined and written in terms of an equivalent displacement-like forcing F. If this combined forcing due to a periodic disturbance translating at constant speed U at angle θ to the x axis is $F=\eta_0 \exp(\mathrm{i}k(x\cos\theta+y\sin\theta-Ut))$, then the steady state forced wave solution to (2.4) is (Lamb 1932; Proudman 1953)

$$\eta_f = \frac{\eta_0}{1-Fr^2}\exp(\mathrm{i}k(x\cos\theta+y\sin\theta-Ut)), \tag{2.5}$$

where the Froude number of the disturbance is $Fr=U/c$. Equation (2.5) shows that the sea-level response is amplified under a moving disturbance. This amplification effect known as 'Proudman resonance' becomes large as $Fr\rightarrow 1$. Like V07, the aspect of (2.5) significant to this work is the dependence of the forced wave amplitude on the water depth, which appears in the denominator of Fr. As the steady state forced wave η_f moves over changes in water depth, transient free waves are generated. These waves are generated by the interaction of the barotropic velocity, associated with the forced wave, with the topography. For topography-oriented parallel to the y axis, (2.1) gives the cross-topographic velocity associated with the forced wave as

$$u_f = \frac{U\eta_0\cos\theta}{h(1-Fr^2)}\exp(\mathrm{i}k(x\cos\theta+y\sin\theta-Ut)). \tag{2.6}$$

Any transient waves reflected by the coast or a topographic step, or transmitted across a step, will be assumed to have the form

$$\eta_r = R\eta_0\exp(\mathrm{i}k_r(-x\cos\theta_r+y\sin\theta_r-c_r t)), \tag{2.7}$$

$$\eta_t = T\eta_0\exp(\mathrm{i}k_t(x\cos\theta_t+y\sin\theta_t-c_t t)), \tag{2.8}$$

where R and T are the reflection or transmission coefficients, k's are the magnitudes of the wavenumbers, c's are the free-wave speeds and subscripts r and t indicate reflected and transmitted waves.

The next two subsections give the transients due to an atmospheric disturbance crossing a vertical coast and a topographic step (figure 1). Some of the solutions presented here are trapped, i.e. have exponential decay in the $-x$ direction. Thus, it is convenient to use periodic solutions in the y direction in order to satisfy (2.4). However, note that the amplitude of the forcing may be a function of the magnitude of the wavenumber, i.e. $\eta_0(k)$, which may be the Fourier transform of a discrete atmospheric event with a dominant wavenumber of $2\pi/L$.

2.1. *Disturbance crossing vertical coast and resonance*

Before deriving the solution for the reflected wave due to an atmospheric disturbance over a flat-bottomed ocean crossing a vertical coast, ray theory will be used to determine the wavenumber and angle of the reflected wave (figure 1*a*). For the phase structures of the forced and reflected waves to match at the coast, their wave frequencies and the components of their wavenumbers parallel to the coast must match, i.e. comparing (2.7) with (2.5) gives $k_r=Fr\,k$ and

$$\sin\theta_r = \frac{\sin\theta}{Fr}. \tag{2.9}$$

For a free wave hitting the coast, the angle of reflection equals the angle of incidence. For a forced wave, the reflection angle obeys a Snell-like law (2.9) due to the differing wave speeds. In this modified reflection law, the angle of reflection is greater than

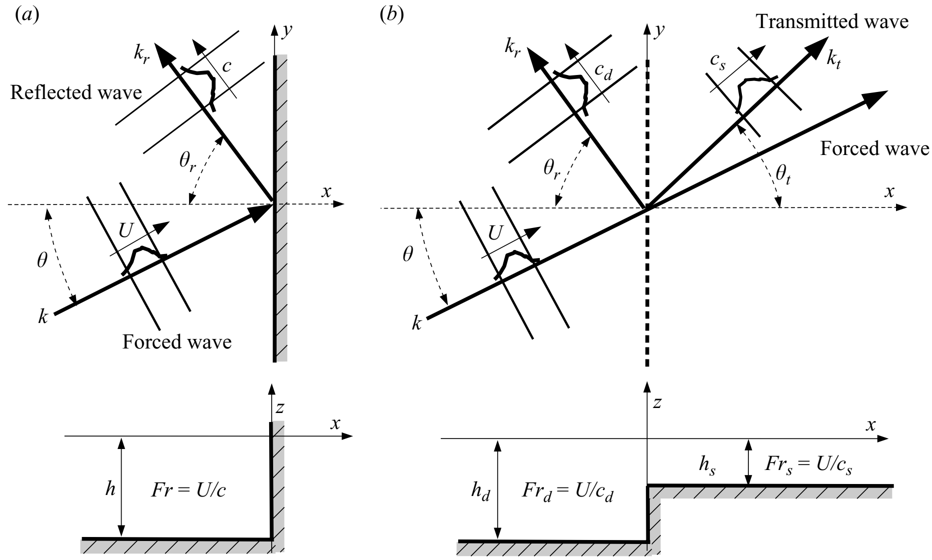

FIGURE 1. Geometry of forced and transient ocean waves due to an atmospheric disturbance obliquely crossing topography. Large arrows give wavenumber vectors making up the disturbance or the free-wave transients. (*a*) Crossing a coast, forced and a reflected free wave. (*b*) Crossing a step, forced wave plus reflected and transmitted free waves. The forced wave grows as it crosses the step because of the enhanced Proudman resonance (2.5).

the incidence angle for subcritical disturbances, $Fr < 1$, and smaller for supercritical disturbances, $Fr > 1$. Another consequence of the modified reflection law is that there is no reflected free wave if θ exceeds a critical angle

$$\theta_{crit} = \sin^{-1} Fr, \tag{2.10}$$

which can only occur for subcritical speed disturbances.

The solution for the reflected wave (2.7), which satisfies the velocity boundary condition that $u_f + u_r = 0$ at $x = 0$, is

$$\eta_r = R\eta_0 \begin{cases} \exp(\mathrm{i}k(-xFr\cos\theta_r + y\sin\theta - Ut)) & \theta < \theta_{crit} \\ \exp(\alpha kx)\exp(\mathrm{i}k(y\sin\theta - Ut) - \mathrm{i}\pi/2) & \theta > \theta_{crit}, \end{cases} \tag{2.11}$$

where

$$R = \frac{Fr^2\cos\theta}{(1 - Fr^2)\sqrt{|Fr^2 - \sin^2\theta|}}, \tag{2.12}$$

where $\alpha = \sqrt{\sin^2\theta - Fr^2}$ and from (2.9) $\cos\theta_r = \sqrt{Fr^2 - \sin^2\theta}/Fr$. There are two types of solutions. For $\theta < \theta_{crit}$, the reflected wave is similar to the forced wave, except that it travels away from the coast with an x wavenumber scaled by $\sqrt{Fr^2 - \sin^2\theta}$. For $\theta > \theta_{crit}$, the reflected wave decays exponentially with scale '$1/\alpha k$' and has wavefronts at right angles to the coast which travel with the forcing. Equation (2.12) contains two resonances: Proudman resonance and the other occurs at the critical incident angle, θ_{crit}. The latter resonance occurs only for subcritical Froude numbers, when the phase speed component of the disturbance along the coast, $U/\sin\theta$, matches the free-wave speed c (2.9). To be valid, the simple linear model requires the reflected wave's amplitude to remain small compared with the water depth. Thus, near θ_{crit}

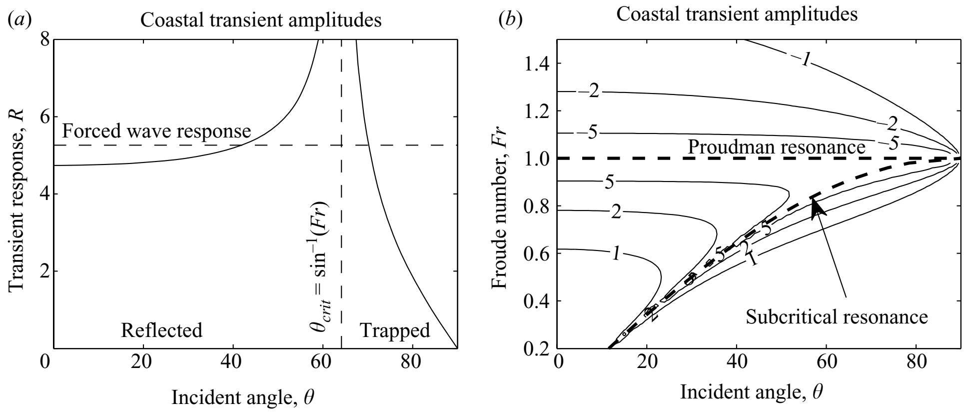

FIGURE 2. Magnitude of the reflection coefficient R from (2.12) for the transient wave generated by an atmospherically forced pressure wave crossing a vertical coast. (*a*) R for $Fr = 0.9$ for a range of disturbance incident angles. (*b*) Contours of R for a range of disturbance incident angles and Froude numbers. The thick dashed lines show the two resonances.

resonance will restrict the application of the model to very weak atmospheric forcing and, strictly, the model cannot be applied at θ_{crit}.

The term R in (2.12) applies to disturbances crossing from the ocean to the land and to those crossing from the land to the ocean. Both propagation directions can produce resonance. Also, while an elevated forced wave generates an elevated reflected wave for a disturbance moving onshore, $0 \leqslant \theta < 90^\circ$, a disturbance moving offshore, $90^\circ < \theta \leqslant 180^\circ$, generates a depressed reflected wave.

Subcritical resonance results from the combination of the boundary condition and the modified reflection law (2.9) giving reflected wave angles greater than the incident angle for $Fr < 1$. The reflected wave's cross-coast velocity is proportional to the x derivative of (2.11), i.e. proportional to $\cos\theta_r$ or $k\alpha$, both of which are zero at θ_{crit}. As the incident angle approaches θ_{crit}, R must increase to balance the decreasing $\cos\theta_r$ or $k\alpha$ in order that their product can give a finite velocity to cancel the forced wave's velocity (2.6) at the coast. Thus to satisfy the boundary condition, R must be singular at θ_{crit}, i.e. resonant. For supercritical speed disturbances, there is no critical angle or resonance and the reflection is always a free wave because the reflection angle (2.9) is less than the incident angle.

Figure 2(*a*) gives the reflection coefficient for $Fr = 0.9$ for which $\theta_{crit} = 64^\circ$. The reflected wave has a similar amplitude to the forced wave up to an incidence angle of 45° and is large around the critical angle. The reflected wave amplitude reduces to zero as the incident angle approaches 90° because the forced wave moving along the coast has zero cross-coast velocity (2.6) and hence requires no reflected wave to satisfy the coastal boundary condition. Figure 2(*b*) shows the reflection coefficient, clearly showing Proudman resonance at $Fr \approx 1$. The lower dashed line shows how subcritical resonance varies with angle of incidence. This figure also shows that the angular width of the region giving a large response widens with increasing Froude number, which makes a large response more likely at high subcritical Froude numbers.

2.2. *Disturbance crossing a step*

When an atmospherically forced wave crosses a step, both a reflected and a transmitted transient free wave are created (figure 1*b*). Again using ray theory, the frequency and

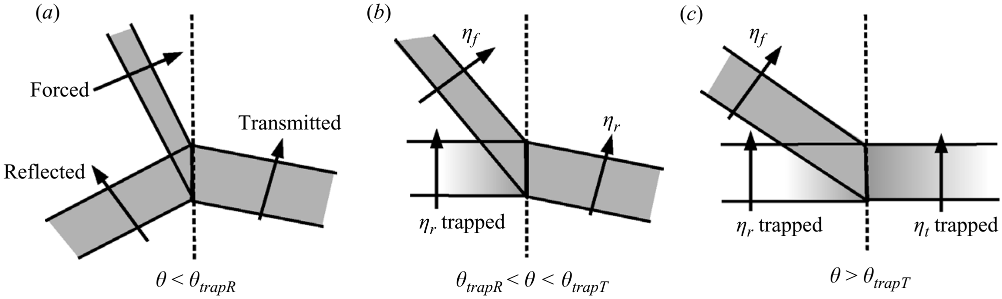

FIGURE 3. Wavefront patterns for a subcritical forced wave crossing a step. There are three cases depending on the angle of incidence. (*a*) Reflected and transmitted waves propagate away as free waves. (*b*) The reflected wave is trapped at the step and the transmitted is a free wave. (*c*) Reflected and transmitted waves are trapped at the step.

the wavenumber parallel to the step of both the reflected and transmitted waves must match those of the forced wave. Thus, from comparing (2.7) and (2.8) with (2.5) gives

$$\sin\theta_r = \frac{\sin\theta}{Fr_d}, \quad \sin\theta_t = \frac{\sin\theta}{Fr_s}, \tag{2.13}$$

i.e. Snell's law reinterpreted for forced waves, where $Fr_d = U/c_d$ and $Fr_s = U/c_s$ are the Froude numbers on each side of the step in figure 1(*b*). The subscripts d and s indicate the deep and shallow sides of the step. Equation (2.13) gives two special angles

$$\theta_{trapR} = \sin^{-1} Fr_d, \quad \theta_{trapT} = \sin^{-1} Fr_s. \tag{2.14}$$

These two angles give three cases for the character of the transient wave pattern for subcritical speed disturbances (figure 3). (i) For small incident angles, $\theta < \theta_{trapR}$, reflected and transmitted waves freely propagate away from the step as in V07. (ii) For $\theta_{trapR} < \theta < \theta_{trapT}$, the reflected wave is a trapped wave decaying exponentially and the transmitted wave freely propagates away. (iii) For $\theta > \theta_{trapT}$, reflected and transmitted waves are both trapped waves with wavefronts normal to the step and decay exponentially with distance from the step.

The reflected wave has the form

$$\eta_r = R\eta_0 \exp(\mathrm{i}k(y\sin\theta - Ut)) \begin{cases} \exp(-\mathrm{i}k\alpha_d x) & \theta \leqslant \theta_{trapR}, \\ \exp(k\alpha_d^* x) & \theta > \theta_{trapR}, \end{cases} \tag{2.15}$$

where $\alpha_d = \sqrt{Fr_d^2 - \sin^2\theta}$ and $\alpha_d^* = \sqrt{\sin^2\theta - Fr_d^2}$. The transmitted wave has the form

$$\eta_t = T\eta_0 \exp(\mathrm{i}k(y\sin\theta - Ut)) \begin{cases} \exp(\mathrm{i}k\alpha_s x) & \theta \leqslant \theta_{trapT}, \\ \exp(-k\alpha_s^* x) & \theta > \theta_{trapT}, \end{cases} \tag{2.16}$$

where $\alpha_s = \sqrt{Fr_s^2 - \sin^2\theta}$ and $\alpha_s^* = \sqrt{\sin^2\theta - Fr_s^2}$. Matching the elevation and mass transport due to the forced and reflected waves on the deep side of the step to the enhanced forced and transmitted waves on the shallow side gives the reflection and

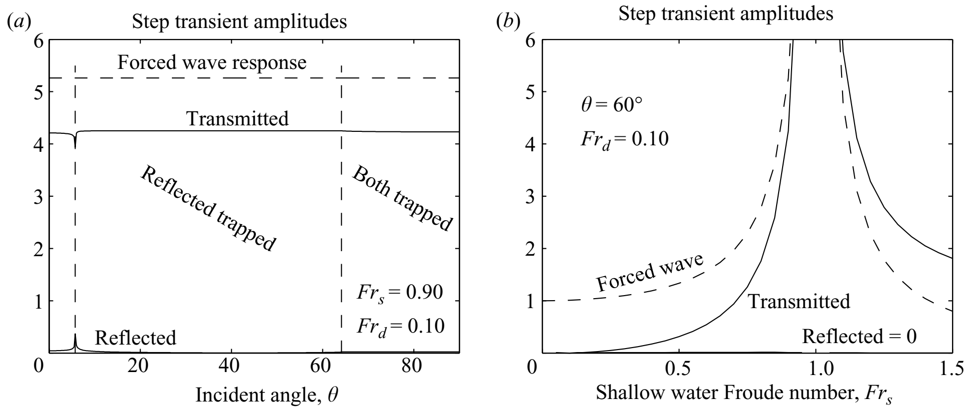

FIGURE 4. Magnitude of the amplitudes of reflected (2.17) and transmitted (2.18) free waves generated by a forced wave crossing a step. Dashed line is the forced wave amplitude on the shallow side (2.5). (*a*) For a range of incident angles and $Fr_s = 0.9$ and $Fr_d = 0.1$. $\theta_{crit} = 64°$. (*b*) For a range of Froude numbers on the shallow side for $\theta = 60°$ and $Fr_d = 0.1$.

transmission coefficients as the amplitudes of

$$R = \Delta Fr_d^2 \frac{\alpha_s - Fr_s^2 \cos\theta}{\alpha_d Fr_s^2 + \alpha_s Fr_d^2}, \tag{2.17}$$

$$T = -\Delta Fr_s^2 \frac{\alpha_d + Fr_d^2 \cos\theta}{\alpha_d Fr_s^2 + \alpha_s Fr_d^2}, \tag{2.18}$$

where

$$\Delta = \frac{Fr_s^2 - Fr_d^2}{\left(1 - Fr_s^2\right)\left(1 - Fr_d^2\right)}, \tag{2.19}$$

which is the fractional difference in forced wave amplitude between shallow and deep water, and also encapsulates Proudman resonance on both sides of the step. Note that reflection and transmission coefficients (2.17) and (2.18) can cover all three cases of wave patterns if α_d and α_s are allowed to be imaginary at larger incident angles. As $h_s \to 0$ in (2.17) $R \to Fr_d \cos(\theta)/(1 - Fr_d^2)\cos\theta_r$, which is the same as for coastal reflection (2.12). The first cases in (2.15)–(2.18) are equivalent to those of Garrett (1970) for a disturbance which has supercritical speed on both sides of the step. He did not explore subcritical disturbances and hence did not look at the trapped cases. The coefficients, (2.17) and (2.18), are given for disturbances crossing from deep to shallow water. The coefficients for disturbances moving from shallow water to deep water have the same form but differ in the signs of some terms, but are not presented here because of space limitations.

Figure 4(*a*) demonstrates that, for small Fr_d, the transmitted wave's amplitude varies little with angle of incidence and the reflected wave amplitude is near zero. Unlike coastal transients, transients are required for $\theta = 90°$ as the differing forced wave amplitudes on the two sides of the step require trapped waves on both sides to ensure continuity of surface elevation across the step. For a small deep water Froude number, $Fr_d \to 0$, (2.17) and (2.18) confirm that for all three cases $R \to 0$ and $T \to -Fr_s^2/(1 - Fr_s^2)$. Thus, the transmitted wave amplitude is approximately Fr_s^2 times that of the enhanced forced wave in shallow water for all incident angles. For the

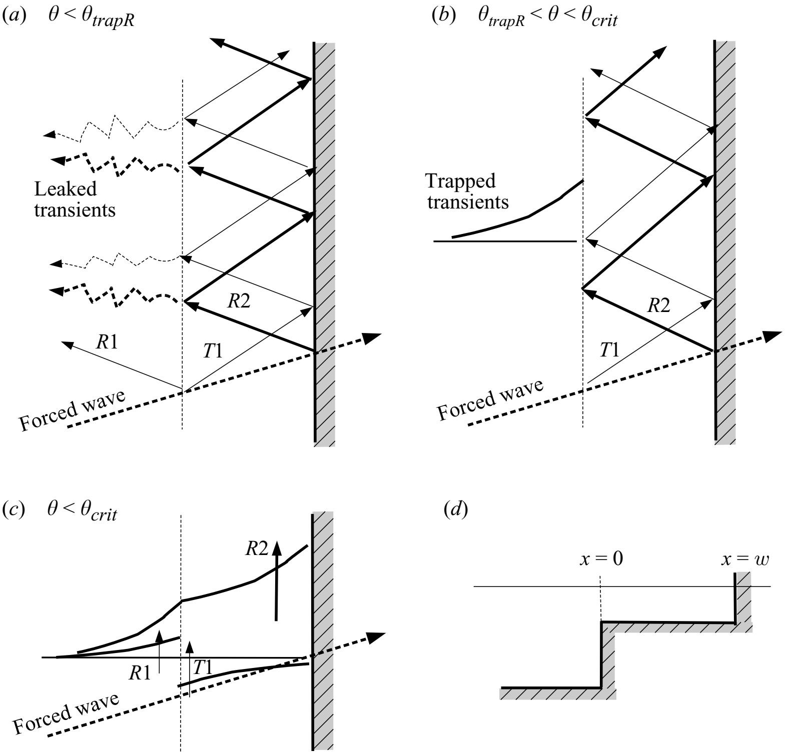

FIGURE 5. Schematic ray paths for the transients generated by a subcritical speed atmospheric pressure disturbance crossing the continental shelf. Note θ_{trapT} is the same as the coastal critical angle θ_{crit}. (a) $\theta < \theta_{trapR}$ transients are leaked. (b) $\theta_{trapR} < \theta < \theta_{crit}$ free-wave transient trapped on shelf by internal reflection. (c) $\theta > \theta_{crit}$ transients are trapped against the step and coast. (d) Idealized topography of the continental shelf.

example in figure 4(a), the forced wave amplitude, (2.5), is around five times the amplitude of the forcing, η_0.

Figure 4(b) shows the amplitudes of the forced and transmitted waves for a range of Froude numbers for one incident angle. However, the transmitted amplitude curve would be almost the same for any incident angle due to the near invariance of T for small Fr_d. For subcritical Fr_s, increasing Froude number increases the size of the transmitted wave relative to the forced wave and they are of similar size around $Fr_s \approx 1$.

3. Discussion

3.1. *Crossing a continental shelf*

The solutions for an atmospheric disturbance crossing a step and vertical coast can be combined to discuss a disturbance crossing the continental shelf. The idealized shelf topography comprises a step change in depth at the continental 'slope', a flat shelf and a vertical coast (figure 5d). Again the results discussed in this section apply to on- and offshore-moving disturbances, with propagation direction affecting only the sign or phase of the transients.

When a disturbance crosses the continental 'slope' and the coast, two separate free-wave transients are generated on the shelf (figure 5a). The fate of these transients depends on the incidence angle of the disturbance. As the disturbance crosses the 'slope', a small transient free wave, $R1$, is reflected back into deep water and a larger free wave, $T1$, is transmitted onto the shelf. Here $T1$ reflects off the coast and from (2.13) impinges on the shelf break at an angle of $\sin^{-1}\sin\theta\, Fr_s^{-1}$. The transient reflected from the disturbance crossing the coast, $R2$, also impinges on the shelf break at this same angle, (2.9). Both $T1$ and $R2$ will each create new transmitted and reflected free waves as they cross the 'slope'. Snell's law for free waves crossing a step into deeper water shows that they will undergo total internal reflection if the incident angle exceeds $\sin^{-1}(c_s/c_d)$ (LeBlond & Mysak 1978). Thus, the free-wave transients on the shelf will be totally internally reflected by the shelf break if the incident angle of the forced wave exceeds the typically small angle

$$\theta \geqslant \sin^{-1} Fr_d = \theta_{trapR}. \tag{3.1}$$

Figure 5 shows the three cases of forced wave incident angles. (i) For small angles, $\theta < \theta_{trapR}$, $T1$ and $R2$ bounce back and forth across the shelf, with a small transient being transmitted into the deep ocean during each encounter with the shelf break, as in V07. These leaked transients progressively reduce the amplitudes of $T1$ and $R2$ as they propagate along the shelf. (ii) For $\theta_{trapR} < \theta < \theta_{crit}$, $T1$ and $R2$ are free waves trapped on the shelf maintaining their amplitudes as they propagate along it (figure 5*b*). As $\theta \to \theta_{trapT} = \theta_{crit}$, $R2$ becomes large because of the coastal resonance, while $T1$'s amplitude remains almost constant. (iii) For $\theta > \theta_{crit}$, $R2$ becomes trapped against the coast (figure 5*c*) and propagates with the forcing. In addition, both $T1$ and the typically small $R1$ are trapped at the step which constitutes the continental slope.

3.2. *Excitation of shelf modes*

Note that $\theta_{crit} = \sin^{-1} Fr_s$ is the incident angle giving resonance for a disturbance crossing the coast, but it is also the angle at which the transients over the shelf, $T1$ and $R2$, change from having sinusoidal to exponential cross-shelf structure. Thus, for $\theta_{trapR} < \theta < \theta_{crit}$, the internally reflected transients over the shelf (figure 5*b*) can be expressed as standing waves and for $\theta > \theta_{crit}$ as a pair of exponentials. For example, the transient $T1$, which satisfies the coastal boundary condition at $x = W$, can be expressed as

$$\eta_{T1} = T\eta_0 \exp(\mathrm{i}k(y\sin\theta - Ut)) \begin{cases} \cos(k\alpha_s(x-W)) & \theta_{trapR} < \theta < \theta_{crit}, \\ \cosh(k\alpha_s^*(x-W)) & \theta \geqslant \theta_{crit}. \end{cases} \tag{3.2}$$

Matching the elevation and mass transport at the step, $x = 0$, of the exponentially decaying reflected wave and the deep water forced wave with this transmitted wave and the enhanced forced wave over the shelf gives the transmission coefficient as the amplitude of

$$T = \Delta Fr_s^2 \begin{cases} \dfrac{\alpha_d^* - \mathrm{i}Fr_d^2\cos\theta}{\alpha_s Fr_d^2 \sin kW\alpha_s - \alpha_d^* Fr_s^2 \cos kW\alpha_s} & \theta_{trapR} < \theta < \theta_{crit} \\ \dfrac{\alpha_d^* - \mathrm{i}Fr_d^2\cos\theta}{-\alpha_s^* Fr_d^2 \sinh kW\alpha_s^* - Fr_s^2\alpha_d^* \cosh kW\alpha_s^*} & \theta \geqslant \theta_{crit}. \end{cases} \tag{3.3}$$

For the step, the transmitted free-wave transient (2.18) is near uniform with incident angle when Fr_d is small and, at high subcritical Fr_s, is comparable in size with the enhanced forced wave on the shallow side of the step (figure 4). In contrast, the standing-wave transient over a shelf (figure 6*a*) can have amplitudes many times

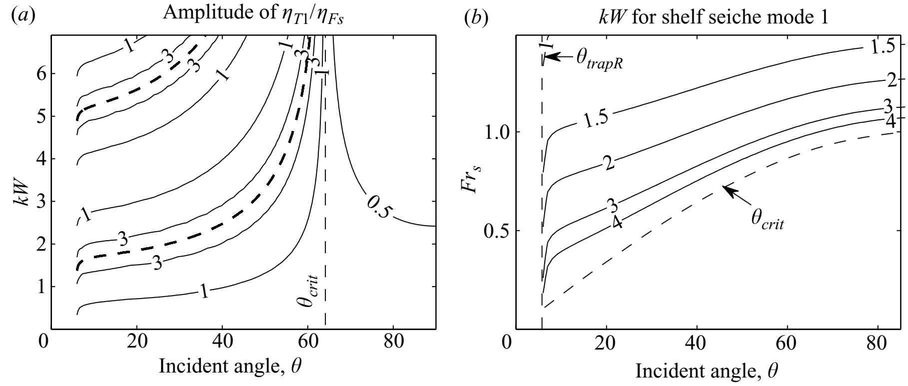

FIGURE 6. Response of a shelf to a forced wave crossing the continental shelf. (*a*) Amplitude of transmitted standing wave transient $T1$, (3.3), relative to the amplitude of the forced wave over the shelf as a function of shelf width and incident angle for $Fr_d = 0.1$ and $Fr_s = 0.9$. The lower thick dashed line represents the width and angle combinations which resonate with a mode 1 shelf seiche, whereas the upper left dashed line represents the width and incident angle combinations which resonate with mode 2. (*b*) Value of kW for a mode 1 shelf seiche when $Fr_d = 0.1$ from (3.4).

larger than those in the forced wave over the shelf for some combinations of incident angles and shelf widths. These combinations between θ_{trapR} and θ_{crit} occur when the forced wave excites a seiche mode of a step shelf, given by Snodgrass, Munk & Miller (1962), via the generation of a transmitted transient $T1$ at the shelf break. Figure 6(*a*) demonstrates that at high incident angles a short wavelength or a wide shelf is required to excite mode 1 and that on wide shelves either mode 1 or mode 2 may be excited depending on the incident angle.

When the denominator of (3.3) vanishes, the forced wave excites shelf modes and its wavenumber must satisfy

$$kW = \frac{1}{\alpha_s}\left(\tan^{-1}\left(\frac{Fr_s^2\alpha_d^*}{Fr_d^2\alpha_s}\right) + n\pi\right), \quad \theta_{trapR} < \theta < \theta_{crit}, \quad n = 0, 1, \ldots \tag{3.4}$$

For $\theta \geqslant \theta_{crit}$, no mode is excited because there is no zero in the denominator of (3.3) in this range, which gives a trapped wave in deep water. The modes resonate when the phase speed of the disturbance along the coast matches the alongshore phase speed of one of Snodgrass *et al.*'s trapped shelf modes. To give a trapped mode on the shelf, Snodgrass *et al.* require the phase speed of the mode along the shelf to lie between c_s and c_d. For an obliquely incident forced wave, this translates into the condition $\theta_{trapR} < \theta < \theta_{crit}$. Figure 6(*b*) shows the value of kW required to excite shelf mode 1. Except near the critical angle, kW is between 1 and 3, so generally disturbances with wavelengths between 2 and 5 times the shelf width can have a magnified response to the forcing, if their incident angle and Froude numbers approximately satisfy (3.4), particularly at the coastal anti-node. Again, like the coastal resonance, the assumed linearity of the model restricts its applicability near the resonant incident angles, and the transient solutions for on- and offshore-moving disturbances differ only in sign.

Figure 7 shows the cross-shelf structure of mode 1. For the two cases with θ just above θ_{trapR}, mode 1 amplitudes reduce by half across the shelf, but in deep water decay is slow for these marginally trapped cases. For the cases with incident angles near θ_{crit}, there is significant decay across the shelf and rapid decay in deep water.

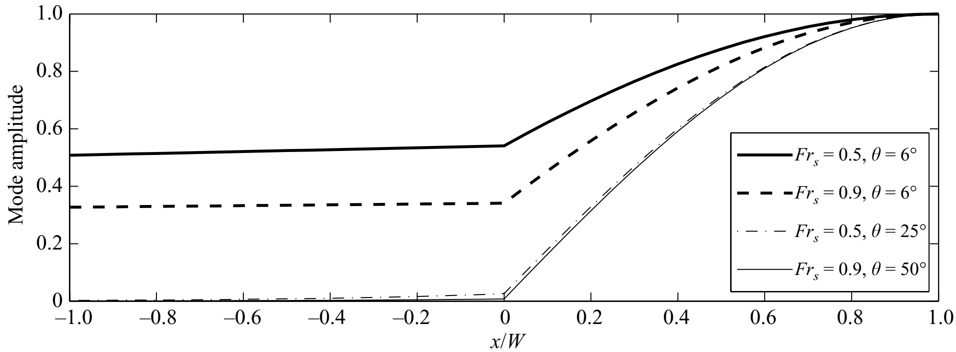

FIGURE 7. Shapes of shelf modes excited by oblique forced pressure wave. For all cases, $Fr_d = 0.1$ and $\theta_{trapR} = 5.7°$. Legend gives shallow water Froude numbers and incident angles. For $Fr_s = 0.5$ cases $\theta_{crit} = 30°$ and for $Fr_s = 0.9$ cases $\theta_{crit} = 64°$.

Thus, modes excited by near critical incident angle disturbances exhibit stronger trapping than those with smaller incident angles.

Greenspan resonance occurs when the translation speed of a point pressure disturbance moving parallel to the coast matches the alongshore speed of a Stokes edge wave mode for a linearly sloping bottom. Here resonant excitation occurs when the alongshore component of the disturbance's phase speed matches the alongshore propagation speed of one of Snodgrass *et al.*'s seiche modes for a step shelf. Thus, the resonant excitation in (3.3) is conceptually equivalent to Greenspan resonance for forced wavefronts traversing a step shelf, but extended to allow for angle of incidence. Note that (3.3) also contains Proudman resonance in the Δ factor, thus the excitation magnifies the effects of the enhanced forced wave over the shelf.

The standing wave form of $T1$ is forced by the disturbance crossing the shelf break. The standing wave $T1$ must satisfy matching conditions at the shelf break and a zero velocity condition at the coast and, consequently, resonates when its cross-shelf structure has an odd number of quarter wavelengths. Although $T1$ is forced at the shelf break, the transient $R2$ (figure 5*b*) is forced at the coast. $R2$ and its reflection from the shelf break can also be expressed as a standing wave for $\theta_{trapR} < \theta < \theta_{crit}$. The standing wave form of $R2$ is matched to the forced wave at the coast and, at the shelf break, to a trapped decaying exponential wave in deep water. The standing wave form of $R2$ can exchange water with this decaying exponential. Hence, a forced wave crossing the coast does not excite shelf modes.

3.3. *Crossing an ocean ridge*

A ridge can be approximated by a parallel pair of back-to-back steps. When a free shallow water wave crosses from deep water onto the ridge, refraction causes a decrease in the angle of propagation. From Snell's law, the maximum transmission angle for the wave on the ridge is $\sin^{-1}(c_s/c_d)$. Thus, the angle with which the transmitted wave on the ridge impinges on the far side of the ridge will generally be less than that required for internal reflection and trapping on the ridge is unlikely. Unlike free waves, the transmission angle of the transient generated by a subcritical speed forced wave crossing onto the ridge is larger than the incident angle, making trapping possible. As for the shelf in the last section, trapping occurs for a forced wave incident at greater than the typically small angle of θ_{trapR}. When a forced wave event due to low atmospheric pressure crosses a ridge at greater than θ_{trapR}, two trapped

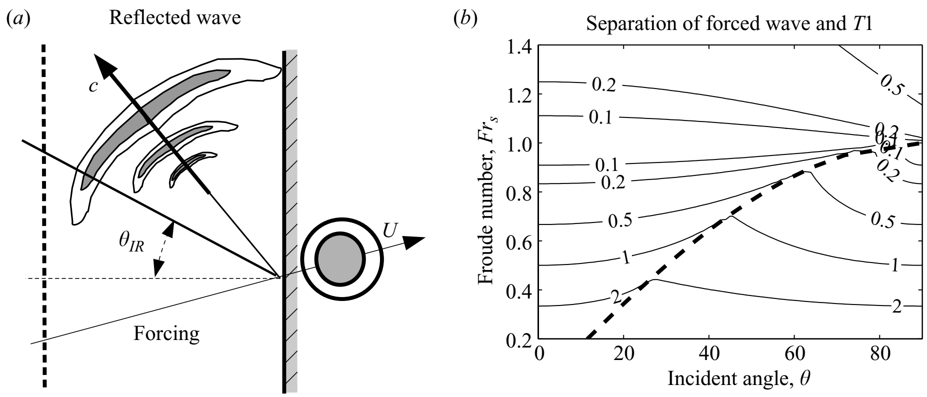

FIGURE 8. (a) Schematic of the amplitude of the reflected transient due to circular disturbance crossing the coast. Here $\theta_{IR} = \sin^{-1}(c_s/c_d)$ is the angle required for internal reflection by the shelf break. (b) Separation between the centre of a finite breadth storm and the transmitted wave $T1$ for a storm crossing the shelf break. Separation is expressed as fraction of the distance the storm moves in one time scale T_p, i.e. its width L. Thick dashed line is θ_{crit}.

free-wave transients propagating along the ridge are generated: a depressive transient from crossing onto the ridge and, trailing slightly behind, an elevated transient from leaving the ridge. For a narrow ridge, the two transients will be generated so close together that they will almost cancel out, giving little trapped energy. Wider ridges will give larger trapped waves up to the limit of T (2.18), i.e. comparable to the enhanced forced wave in shallow water for high subcritical disturbance speeds (2.5). Like $R2$ on the shelf, the ability of the internally reflected transients to exchange fluid with a trapped deep water wave means a forced wave crossing a ridge also does not excite seiche modes of the ridge.

3.4. *Finite breadth disturbances*

The solutions presented here are for forced waves with infinitely broad wavefronts, so are essentially rotated one-dimensional solutions. The two-dimensional transients from a subcritical circular point disturbance spread in all directions away from the location where the disturbance crosses the coast or shelf break. However, amplitudes are higher in the direction of their propagation (figure 8a) as demonstrated by the two-dimensional numerical solutions in V07, and thus transients have a preferred direction given by their ray paths. Radial spread influences the degree to which the energy of the transients is trapped on the shelf or ridge. Transients from a circular disturbance crossing a shelf will impinge on the shelf break at a range of incident angles, some less than the angle required for internal reflection (figure 8a). Thus, with successive reflections from the shelf break and coast, the transients will leak energy into deep water, decaying as they progress along the shelf. As the forced wave's angle of incidence increases towards the critical angle, the transients' preferred directions become more aligned with the shelf and more of the spreading transient energy will be internally reflected back onto the shelf. More work is required to quantify the trapping of transient energy due to finite breadth disturbances, potentially using Greens functions; however, trapping will be the strongest for disturbances with incident angles at and beyond the critical angle.

3.5. *Storm surge*

A subcritical storm crossing a steep coast near the critical angle will generate a large elevated transient, $R2$, travelling nearly parallel to the coast and trapped on the shelf, resulting in transient wave energy, which can enhance storm surge heights locally and may impact distant coastal areas with long period waves.

The transmitted transient $T1$ also plays a role in storm surge by influencing how quickly an atmospheric-pressure-induced surge develops over the shelf. Proudman resonance is enhanced as a storm moves from deep water onto the shelf. This enhancement does not occur instantaneously after crossing the shelf break, but develops slowly as the forced wave and the transmitted wave $T1$ separate because of their differing speeds and propagation directions. V07 discussed separation for storms crossing the shelf at right angles. A measure of the rate of separation is the distance between the centre of a circular storm and the centre of the transmitted free wave after crossing the shelf break. Separation can also be viewed in another way. As a particular section of an oblique forced wave's crest crosses the shelf break, it creates a transmitted wave. Separation is the distance between the section of the forced wave's crest which generated the transient and the section of the transient it generated. In other words, separation is the distance between the points on both waves which were at the shelf break at the same time and place. The components of this separation are

$$(\Delta x,\, \Delta y)/L = \left(\cos\theta - \frac{\cos\theta_t}{Fr_s},\ \sin\theta - \frac{\sin\theta_t}{Fr_s}\right), \tag{3.5}$$

where the separation is expressed as a fraction of the storm width, L, i.e. the distance travelled by the storm in one time scale T_p. Note that θ_t is given by (2.13) for $\theta < \theta_{crit}$ and is 90° for $\theta > \theta_{crit}$. This measure (3.5) underestimates the rate at which the enhanced Proudman resonance develops for point disturbances, because of radial spread of $T1$, but is a useful relative measure. Figure 8(*b*) shows the magnitude of the separation for a range of incident angles and Froude numbers. Separation, and hence enhancement of Proudman resonance, develops more quickly for very slow or very fast disturbances, because the speed differential between storm and transient is large. For example, for $Fr_d = 0.1$ and $Fr_s = 0.4$, by the time the storm has moved one width the transient is two widths away and the enhanced Proudman resonance on the shelf is nearly fully developed. However, from (2.19), the enhancement of Proudman resonance over the shelf for this case is only 18 %. For high subcritical Fr_s, the enhancement is much larger, e.g. around 400 % for $Fr_d = 0.1$ and $Fr_s = 0.9$, but takes longer to develop. Given that shelf widths may be comparable to the diameter of a small storm, the enhanced Proudman resonance may not have time to fully develop before the storm crosses the coast. Hence, shelf width will limit the storm surge associated with atmospheric pressure. The significant aspect of figure 8(*b*) is that for high subcritical Fr_s, the separation occurs more rapidly for storms crossing the shelf break near the critical angle, θ_{crit}. Consequently, even without subcritical resonance, surge at the coast is larger under fast subcritical storms which cross narrow shelves near this particular angle.

4. Conclusions

The reflection and refraction laws for forced waves determine how the incident angle of an atmospheric disturbance influences the fates of the transients generated as it crosses topography. In particular, for subcritical speed disturbances the angle of reflection at a coast and the transmission angle at a step are larger than the incident angle. As a result, (*a*) the coastally reflected wave becomes resonant at $\theta_{crit} = \sin^{-1} Fr_s$, and at greater incident angles it is trapped; and (*b*) the coastally reflected transient, $R2$, and the transients transmitted onto a shelf or a ridge, $T1$, are trapped for forced waves incident at angles greater than the typically small $\theta_{trapR} = \sin^{-1} Fr_d$. Consequently, for $\theta_{trapR} < \theta < \theta_{crit}$, a shelf or a ridge can act as a waveguide for relativity high frequency sea-level energy derived from atmospheric disturbances, which may excite seiches within inlets or on the shelf. For point atmospheric disturbances, the degree to which transient energy is trapped within the waveguide increases as the angle of incidence increases towards the critical angle.

Like Proudman resonance, subcritical resonance is independent of the forcing's wavenumber, hence gives the same magnification both for periodic pressure disturbances and for discrete atmospheric events. A storm crossing a coast near the critical angle can generate a large transient travelling along the shelf, enhancing the resulting storm surge. Even in the absence of subcritical resonance, the transient which is generated at the continental shelf break can also affect the magnitude of the storm surge. The enhanced Proudman resonance over a shallow shelf takes time to develop, because the forced and transmitted waves must separate. This separation occurs more rapidly near the critical angle of incidence, enhancing surges at the coast for storms crossing narrow shelves near this angle.

In two situations the effects of the enhanced Proudman resonance over the shelf are further magnified by transients generated by disturbances incident at particular angles. Firstly, a forced wave crossing the coast at the critical angle can cause a resonance via the generation of $R2$. Secondly, a forced wave crossing the shelf break at particular angles between θ_{trapR} and θ_{crit} can excite resonant modes of the shelf via the generation of $T1$. This excitation is conceptually equivalent to Greenspan resonance, extended to allow for the angle of incidence. Because of these seiche modes, the spectral components of the forced wave crossing the shelf, which approximately satisfy (3.4), may contribute to seiche energy over the shelf, which persists long after the disturbance has crossed the coast and in particular may enhance sea-level variability at the coastal anti-node.

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